# Non-triviality and the tuning of the scaling parameter in scale-covariant $\lambda_0(\varphi^2)^2$ field theory



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# Non-triviality and the tuning of the scaling parameter in scale-covariant $\lambda_0(\varphi^2)^2$ field theory

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**Abstract.** We generalise the results of Rivers on an O($N$)-symmetric scale-covariant pseudo-free theory by considering the case of an explicit $\lambda_0(\varphi^2)^2$ interaction term in the Lagrangian. We find that for general values of the scaling parameter $\beta$, the theory is trivial. However, in four and five dimensions there is a 'tuned' form for $\beta$ at which the theory may be non-trivial. In this case, the theory reduces to the pseudo-free theory (which has the same form as the canonical $\lambda(\varphi^2)^2$ theory to leading order). It thus appears that the 'hard-core' effects of scale-covariant quantisation act as a $\lambda\varphi^4$-type interaction.

## 1. Introduction

Recent rigorous results (Aizenman 1981, Fröhlich 1982) show that a canonical scalar field theory with a $\lambda\varphi^4$-type interaction is trivial in $d>4$ dimensions, and it is widely believed that these results also apply to the case of $d=4$ dimensions. However, if the theory is quantised using the scale-covariant methods of Klauder (1981), the inequalities used in the proof of the triviality of the canonical theory are no longer valid, and it is at least plausible that this non-canonical theory is non-trivial.

In a recent paper (Rivers 1983) it was shown that the large-$N$ limit of a scale-covariant O($N$)-symmetric scalar theory with no $\lambda\varphi^4$ interaction can be reduced to a non-trivial pseudo-free theory in $d=4$ and $d=5$ dimensions by a specific tuning of the scaling parameter $\beta$ that characterises the scale covariance. In particular, in $d=4$ dimensions, $\beta$ has to be renormalised (this is not unexpected in the sense that it may be thought of as a coupling constant), and as the regularisation is removed, it was found that the only consistent value of $\beta$ for a non-trivial theory is zero. This does not, however, imply that the theory is the same as the canonical theory, since the tuning is made before the regularisation is removed. The non-triviality of this theory can be explained in terms of the dynamics of the fields, and it was found that for the particular 'tuned' values of $\beta$, bound states and/or resonances occur which mediate the interaction, while for general values of $\beta$ these states do not propagate and hence the theory is trivial.

In this paper we generalise these results to a scale-covariant O($N$)-symmetric scalar theory with a $\lambda(\varphi^2)^2$-type interaction term.

In § 2 we introduce the model and indicate how a saddle-point development may be used to obtain a $1/N$ expansion for the effective potential. Retaining only leading-order terms in this expansion, we obtain the scale-covariant mass-gap equation.

In § 3 we consider the renormalisation of the mass-gap equation. The cases of $d=4$ and $d=5$ dimensions are treated separately, and in each of these cases it is shown how the scaling parameter $\beta$ must be tuned if the theory is to be non-trivial.

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Section 4 indicates briefly how the triviality of the theory may be understood in terms of the dynamics of the fields.

In § 5 we summarise our findings and comment on their implications regarding a recent Monte Carlo study of scale-covariant field theories.

## 2. The model

The theory we wish to describe is that of a set of $N$ scalar fields $\varphi_i$ ($i = 1, \ldots, N$). The Lagrangian density is given by

$$\mathscr{L} = \tfrac{1}{2}(\partial_\mu \boldsymbol{\varphi})^2 + \tfrac{1}{2}m_0^2\boldsymbol{\varphi}^2 + (\lambda_0/8N)(\boldsymbol{\varphi}^2)^2 \qquad \left(\text{where } \boldsymbol{\varphi}^2 = \sum_i \varphi_i\varphi_i\right) \tag{1}$$

and is seen to be O($N$) invariant.

Following Klauder (1981) we generalise the canonical Green function generating functional by using a 'scale-covariant' measure $\tilde{\mathscr{D}}[\boldsymbol{\varphi}]$ such that under

$$\boldsymbol{\varphi}(x) \to S(x)\boldsymbol{\varphi}(x) \qquad : S(x) > 0 \tag{2}$$

$\tilde{\mathscr{D}}[\boldsymbol{\varphi}]$ transforms covariantly,

$$\tilde{\mathscr{D}}[\boldsymbol{\varphi}] \to F[S(x)]\tilde{\mathscr{D}}[\boldsymbol{\varphi}]. \tag{3}$$

Thus, the scale-covariant generating functional is given by

$$Z[\boldsymbol{j}] = \int \tilde{\mathscr{D}}[\boldsymbol{\varphi}] \exp \frac{-1}{\hbar} \int \mathrm{d}x(\mathscr{L} - \boldsymbol{j} \cdot \boldsymbol{\varphi}). \tag{4}$$

Klauder has shown that the most general homogeneous form for the scale-covariant measure $\tilde{\mathscr{D}}[\boldsymbol{\varphi}]$ can be written formally in terms of the canonical translation-invariant measure $\mathscr{D}[\boldsymbol{\varphi}]$ as

$$\tilde{\mathscr{D}}[\boldsymbol{\varphi}] = \tilde{\mathscr{D}}_\beta[\boldsymbol{\varphi}] := \prod_x \frac{\mathrm{d}\boldsymbol{\varphi}_x}{|\boldsymbol{\varphi}_x|^\beta} = \exp\left(\frac{-\beta N}{2}\delta(0) \int \mathrm{d}x \ln \frac{\boldsymbol{\varphi}^2}{N}\right)\mathscr{D}[\boldsymbol{\varphi}] \tag{5}$$

and hence

$$Z[\boldsymbol{j}] = \int \mathscr{D}[\boldsymbol{\varphi}] \exp \frac{-1}{\hbar} \int \mathrm{d}x\{\tfrac{1}{2}(\partial_\mu \boldsymbol{\varphi})^2 + \tfrac{1}{2}m_0^2\boldsymbol{\varphi}^2 + (\lambda_0/8N)(\boldsymbol{\varphi}^2)^2 + \tfrac{1}{2}\beta N\hbar\, \delta(0) \ln(\boldsymbol{\varphi}^2/N) - \boldsymbol{j} \cdot \boldsymbol{\varphi}\}. \tag{6}$$

Using the identities

$$\delta[\boldsymbol{\varphi}^2 - N\rho] = \int \mathscr{D}[\alpha] \exp \frac{-\mathrm{i}}{\hbar} \int \mathrm{d}x\, \alpha(\boldsymbol{\varphi}^2 - N\rho) \tag{7}$$

and

$$\exp \frac{-1}{\hbar} \int \mathrm{d}x\left(\frac{\lambda_0}{8N}(\boldsymbol{\varphi}^2)^2 + \frac{\beta N\hbar}{2}\delta(0) \ln \frac{\boldsymbol{\varphi}^2}{N}\right) = \int \mathscr{D}[\rho]\, \delta[\boldsymbol{\varphi}^2 - N\rho] \exp \frac{-1}{\hbar} \int \mathrm{d}x\left(\frac{\lambda_0 N}{8}\rho^2 + \frac{\beta N\hbar}{2}\delta(0) \ln \rho\right) \tag{8}$$

we can write the scale-covariant generating functional as

$$Z[\boldsymbol{j}] = \int \mathscr{D}[\boldsymbol{\varphi}]\mathscr{D}[\alpha]\mathscr{D}[\rho] \exp \frac{-1}{\hbar} \int \mathrm{d}x \{\tfrac{1}{2}\boldsymbol{\varphi} \cdot (-\nabla^2 + m_0^2 + \mathrm{i}\alpha)\boldsymbol{\varphi} - \boldsymbol{j} \cdot \boldsymbol{\varphi} + \tfrac{1}{2}\mathrm{i} N \alpha \rho + \tfrac{1}{2}\beta N \hbar\, \delta(0) \ln \rho + \tfrac{1}{8}\lambda_0 N \rho^2\} \tag{9}$$

where $\alpha$ and $\rho$ may be thought of as auxiliary fields.

This form is now Gaussian in the $\varphi_i$ fields, so by defining $\chi = m_0^2 + \mathrm{i}\alpha$ and integrating over the $\varphi_i$ fields we have

$$Z[\boldsymbol{j}] = \int \mathscr{D}[\chi]\mathscr{D}[\rho] \exp(-N/\hbar)\mathfrak{A}[\rho, \chi; \boldsymbol{j}] \tag{10}$$

where

$$\mathfrak{A}[\rho, \chi; \boldsymbol{j}] = -(2N)^{-1} \int \mathrm{d}x\, \mathrm{d}y\, (\boldsymbol{j}(x) \cdot G(x, y; \chi)\boldsymbol{j}(y)) + \tfrac{1}{2}\hbar\, \mathrm{Tr} \ln(-\nabla^2 + \chi) + \int \mathrm{d}x \{\tfrac{1}{2}\beta\hbar\, \delta(0) \ln \rho - \tfrac{1}{2}\rho(\chi - m_0^2) + \tfrac{1}{8}\lambda_0 \rho^2\} \tag{11}$$

and

$$(-\nabla_x^2 + \chi(x))G(x, y; \chi) = \delta(x - y). \tag{12}$$

Since each term in $\mathfrak{A}$ is O(1) (Coleman *et al* 1974), a saddle point development may be used to obtain a $1/N$ expansion for $Z[\boldsymbol{j}]$. If only leading-order terms in this expansion are kept, we find an effective potential $\mathscr{V}(\boldsymbol{\varphi}) = \mathscr{V}(\boldsymbol{\varphi}, \chi_0(\boldsymbol{\varphi}^2), \rho_0(\boldsymbol{\varphi}^2))$ with

$$\mathscr{V}(\boldsymbol{\varphi}, \chi, \rho) = \tfrac{1}{2}\chi\boldsymbol{\varphi}^2 - \tfrac{1}{2}N\rho(\chi - m_0^2) + \tfrac{1}{2}\beta N\hbar\, \delta(0) \ln \rho + \tfrac{1}{8}\lambda_0 N \rho^2 + \tfrac{1}{2}N\hbar \int \text{đ}k\, \ln(k^2 + \chi) \tag{13}$$

and

$$\frac{\partial}{\partial \chi}\mathscr{V}(\boldsymbol{\varphi}, \chi, \rho)\bigg|_{\substack{\chi = \chi_0(\boldsymbol{\varphi}^2) \\ \rho = \rho_0(\boldsymbol{\varphi}^2)}} = \frac{\partial}{\partial \rho}\mathscr{V}(\boldsymbol{\varphi}, \chi, \rho)\bigg|_{\substack{\chi = \chi_0(\boldsymbol{\varphi}^2) \\ \rho = \rho_0(\boldsymbol{\varphi}^2)}} = 0,$$

i.e.

$$\rho_0 = \boldsymbol{\varphi}^2/N + \hbar G(\chi_0), \qquad \chi_0 = m_0^2 + \tfrac{1}{2}\lambda_0\rho_0 + \beta\hbar\, \delta(0)/\rho_0. \tag{14}$$

Equations (14) are the scale-covariant equivalent of the mass-gap equation in the Hartree–Fock approximation. They may be represented diagrammatically (Ebbutt and Rivers 1982) as in figure 1.

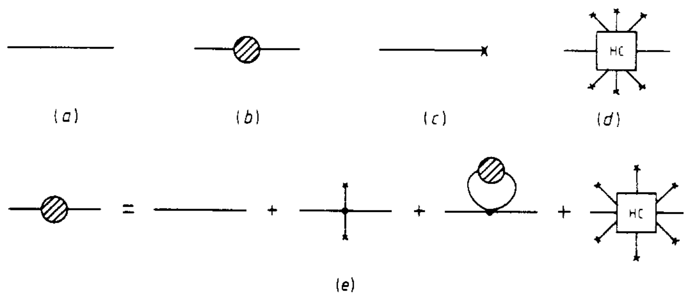


**Figure 1.** The bare $\chi$-propagator with pole at $m_0$ is represented in ($a$), the full $\chi$-propagator in ($b$), the coupling of the $\chi$-field to the $\boldsymbol{\varphi}$-fields in ($c$), and the non-local 'hard-core' interaction in ($d$). The mass-gap equations (14) are then represented in ($e$).

The hard-core interaction is of a non-polynomial form and is highly singular. However, it can be shown that in the $1/N$ expansion these singularities can be absorbed by renormalisation in $d=4$ and $d=5$ dimensions.

The unrenormalised mass-gap equation is seen from (14) to be

$$\chi_0(\varphi^2)=m_0^2+\tfrac{1}{2}\lambda_0(\varphi^2/N+\hbar G(\chi_0))+\beta\hbar\,\delta(0)(\varphi^2/N+\hbar G(\chi_0))^{-1} \tag{15}$$

to leading order in $1/N$.

Since $\partial\mathscr{V}/\partial\varphi^2=2\chi_0(\varphi^2)$, it follows that in order to renormalise $\mathscr{V}$ we must renormalise $\chi_0$.

## 3. Renormalisation

So far our treatment has been independent of the dimension of space–time. However, the renormalisation turns out to be dimension dependent and from now on, we treat $d=4$ and $d=5$ dimensions separately.

*$d=5$ dimensions*

If we use a simple momentum cut-off form of regularisation such that $|k|<\Lambda$, then

$$G_\Lambda(\chi_0):=\int_{|k|<\Lambda}\mathrm{d}^5k\,\frac{1}{k^2+\chi_0}\xrightarrow[\Lambda\to\infty]{}\left(\frac{\Lambda^3}{3}-\chi_0\Lambda+\frac{\pi}{2}\chi_0^{3/2}\right)S_5 \tag{16}$$

and

$$\delta(0):=\int_{|k|<\Lambda}\mathrm{d}^5k=\tfrac{1}{5}\Lambda^5S_5 \tag{17}$$

where $S_5=(2\pi)^{-5}\times$surface area of unit 4-sphere.

Using these regularised forms, and expanding $(\varphi^2/N+\hbar G(\chi_0))^{-1}$ in negative powers of $\Lambda$, the mass-gap equation (15) becomes

$$\begin{aligned}\chi_0[1-\beta\tfrac{9}{5}+\tfrac{1}{2}\lambda_0\Lambda\hbar S_5]&\\ =\left(m_0^2+\beta\tfrac{3}{5}\Lambda^2+\lambda_0\frac{\hbar S_5}{6}\Lambda^3\right)&+\left(\lambda_0-\frac{18}{5}\frac{1}{\hbar S_5}\frac{\beta}{\Lambda}\right)\frac{1}{2}\varphi^2/N\\ &+\left(\lambda_0-\frac{18}{5}\frac{1}{\hbar S_5}\frac{\beta}{\Lambda}\right)\frac{\hbar S_5\pi}{4}\chi_0^{3/2}+\mathrm{O}(\Lambda^{-2}).\end{aligned} \tag{18}$$

Now, since $\beta$ can be thought of as a coupling constant, we expect to have to renormalise it. Thus we introduce the regularised form $\beta_\Lambda$. In order to renormalise (18), we must choose $\beta_\Lambda$, $\lambda_0$ and $m_0$ as functions of $\Lambda$ such that all physical quantities remain finite in the limit $\Lambda\to\infty$.

It will be seen that if we choose $\beta_\Lambda$, $\lambda_0$ and $m_0$ such that

$$1-\beta_\Lambda\tfrac{9}{5}+\tfrac{1}{2}\lambda_0\Lambda\hbar S_5=\alpha+M/\Lambda, \tag{19}$$

where $\alpha$ is an arbitrary dimensionless parameter and $M$ is an arbitrary parameter with the dimension of mass, then (18) becomes

$$\chi_0 \frac{\hbar S_5 \Lambda}{2} \frac{(M/\Lambda+\alpha)}{(M/\Lambda+\alpha-1)} = \frac{\hbar S_5\Lambda}{2} \frac{(m_0^2+\beta_\Lambda \frac{3}{5}\Lambda^2+\lambda_0 \frac{1}{6}\hbar S_5\Lambda^3)}{(M/\Lambda+\alpha-1)} + \tfrac{1}{2}\varphi^2/N + \tfrac{1}{4}\hbar S_5 \pi \chi_0^{3/2} + \mathrm{O}(\Lambda^{-1}). \tag{20}$$

Two separate cases are now seen to be of interest. These are the cases $\alpha = 0$ and $\alpha \neq 0$ when the left-hand side of (20) is $\mathrm{O}(\Lambda^0)$ and $\mathrm{O}(\Lambda^1)$ respectively. We treat these cases independently.

*$\alpha \neq 0$*

From (20) we see that

$$\chi_0 \alpha = (m_0^2 + \beta_\Lambda \tfrac{3}{5}\Lambda^2 + \tfrac{1}{6}\lambda_0 \hbar S_5 \Lambda^3) + \mathrm{O}(\Lambda^{-1}). \tag{21}$$

Now if we renormalise $m_0^2$ such that

$$m_0^2 + \beta_\Lambda \tfrac{3}{5}\Lambda^2 + \tfrac{1}{6}\lambda_0 \hbar S_5 \Lambda^3 = \alpha m^2,$$

where $m$ is an arbitrary parameter with the dimension of mass, then

$$\chi_0 = m^2 \tag{22}$$

and it is seen that $\chi_0$ is independent of $\varphi^2$, and thus to leading order in $1/N$, the effective potential $\mathscr{V}(\varphi^2)$ is given by

$$\mathscr{V}(\varphi^2) = m^2\varphi^2 + \text{constant}.$$

The theory thus appears to describe a free theory for arbitrary $\alpha \neq 0$.

*$\alpha = 0$*

When $\alpha = 0$, the left-hand side of (20) is $\mathrm{O}(\Lambda^0)$, the same order as the terms in $\varphi^2/N$, and thus $\chi_0$ is dependent on $\varphi^2/N$.

More explicitly

$$-\tfrac{1}{2}\hbar S_5 M \chi_0 = -\tfrac{1}{2}\hbar S_5\, \Lambda(m_0^2 + \lambda_0 \tfrac{1}{3}\hbar S_5\, \Lambda^3 + \tfrac{1}{3}\Lambda^2 - M\Lambda) + \tfrac{1}{2}\varphi^2/N + \tfrac{1}{4}\hbar S_5 \pi \chi_0^{3/2} + \mathrm{O}(\Lambda^{-1}) \tag{23}$$

which, by a suitable renormalisation of $\lambda_0$ and $m_0$, can be written as

$$\chi_0/\lambda_R = m_R^2/\lambda_R + \tfrac{1}{2}\varphi^2/N + \tfrac{1}{4}\hbar S_5 \pi \chi_0^{3/2} \tag{24}$$

where $\lambda_R$ and $m_R$ are finite quantities.

Equation (24) is identical to the mass-gap equation for the pseudo-free theory in $d = 5$ dimensions which in turn is identical to that for the canonical theory in $d = 5$ dimensions (Rembiesa 1978). This does not, however, imply that the theory will be trivial, as is the canonical theory in $d = 5$ dimensions, since the derivation of (24) follows a different path from that of the canonical mass-gap equation. Indeed, we have seen that this non-trivial behaviour can occur only when $\alpha = 0$, and from (19) it will be seen that this restricts $\beta_\Lambda$ such that

$$\beta_\Lambda = \tfrac{5}{9}(1 + \lambda_0 \Lambda \hbar S_5/2 - M/\Lambda). \tag{25}$$

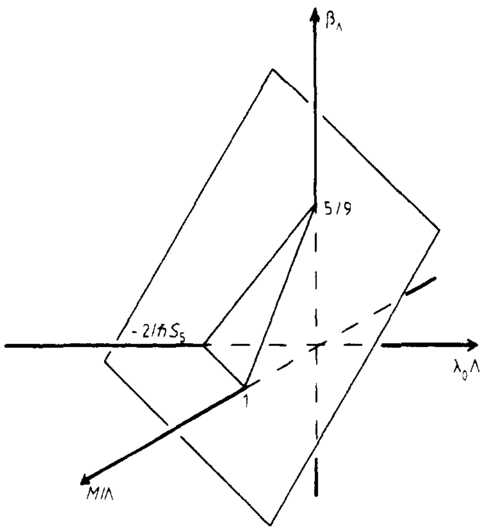


**Figure 2.** In five dimensions, the parameters $\beta_\Lambda$, $M$ and $\lambda_0$ of a non-trivial theory are restricted to the plane $\beta_\Lambda = \frac{5}{9}(1 + \lambda_0\Lambda\hbar S_5/2 - M/\Lambda)$.

For the regularised non-trivial theories, $\beta_\Lambda$ is restricted to the plane shown in figure 2.

As the cut-off is removed ($\Lambda \to \infty$), $M/\Lambda \to 0$, and $\beta_\Lambda$ is further restricted to the line $\beta_\Lambda = \frac{5}{9}(1 + \lambda_0\Lambda\hbar S_5/2)$. This result is in agreement with the work of Rivers (1983) where $\beta_\Lambda = \frac{5}{9}(1 - \mathrm{M}/\Lambda)$ was shown to be the only value of $\beta_\Lambda$ consistent with a non-trivial pseudo-free theory.

In summary we see that, in the large-$N$ limit, there is no difference in $d = 5$ dimensions between the renormalised scale-covariant $\mathrm{O}(N)$-invariant pseudo-free (i.e. $\lambda_0 \equiv 0$) scalar theory and the renormalised scale-covariant $\lambda_0(\varphi^2)^2$ scalar theory: the 'hard-core' effects of scale-covariant quantisation behave as a $\lambda\varphi^4$-type interaction.

*d = 4 dimensions*

We will now consider the theory in four dimensions. Using the same momentum cut-off as before, we have

$$G_\Lambda(\chi_0) := \int_{|k|<\Lambda} \mathrm{d}^4 k \frac{1}{k^2 + \chi_0} = \left(\frac{\Lambda^2}{2} - \frac{\chi_0^2}{2} \ln \frac{\Lambda^2 + \chi_0}{\chi_0}\right) S_4 \tag{26}$$

and

$$\delta_\Lambda(0) := \int_{|k|<\Lambda} \mathrm{d}^4 k = \tfrac{1}{4}\Lambda^4 S_4 \tag{27}$$

with $S_4 = (2\pi)^{-4} \times$ surface area of unit 3-sphere.

Substituting these expressions in (15) and expanding in negative powers of $\Lambda$, we find

$$\chi_0\left[1 + \left(\frac{\lambda_0 \hbar S_4}{4} - \frac{\beta}{2}\right) \ln \frac{\Lambda^2 + \chi_0}{\chi_0}\right]$$
$$= \left(m_0^2 + \beta\frac{\Lambda^2}{2} + \lambda_0 \frac{\hbar S_4}{4}\Lambda^2\right) + \frac{2}{\hbar S_4}\left(\lambda_0 \frac{\hbar S_4}{4} - \frac{\beta}{2}\right)\frac{\varphi^2}{N} + \mathrm{O}(\Lambda^{-2}). \tag{28}$$

In order that the renormalised $\chi_0$ be dependent on $\varphi^2$ (and hence the resulting theory be non-trivial) we must regularise $\beta = \beta_\Lambda$ in such a way that

$$\left[1+\left(\lambda_0\frac{\hbar S_4}{4}-\frac{\beta_\Lambda}{2}\right)\ln\frac{\Lambda^2+\chi_0}{\chi_0}\right]\left(\lambda_0\frac{\hbar S_4}{4}-\frac{\beta_\Lambda}{2}\right)^{-1} \tag{29}$$

remains finite as $\Lambda\to\infty$.

It can readily be seen that if we choose $\beta_\Lambda$ such that†

$$(\lambda_0\tfrac{1}{4}\hbar S_4-\tfrac{1}{2}\beta_\Lambda)^{-1}+\ln(\Lambda^2/M^2)=1/g \tag{30}$$

and $m_0$ such that

$$(\lambda_0\tfrac{1}{4}\hbar S_4-\tfrac{1}{2}\beta_\Lambda)^{-1}(m_0^2+\beta_\Lambda\tfrac{1}{2}\Lambda^2+\lambda_0\tfrac{1}{4}\hbar S_4\Lambda^2)=m^2/g \tag{31}$$

where $M$ is an arbitrary parameter with dimension of mass, $g$ is a dimensionless parameter and $m$ is a massive parameter, then (28) becomes

$$\chi_0/g-\chi_0\ln(\chi_0/M^2)-m^2/g=(2/\hbar S_4)\varphi^2/N. \tag{32}$$

This equation is identical to the pseudo-free and canonical mass-gap equations in four dimensions (Coleman *et al* 1974).

We see from (30) that a non-trivial theory is obtained as the cut-off is removed only if

$$\beta_\Lambda\searrow\tfrac{1}{2}\hbar S_4\,\lambda_0. \tag{33}$$

This is consistent with the work of Rivers on the pseudo-free theory where it was found that in four dimensions a non-trivial solution can be obtained only if $\beta_\Lambda\searrow 0$.

Thus, it appears that in $d=4$ dimensions also, the 'hard-core' effects of scale-covariant quantisation behave as a $\lambda\varphi^4$-type interaction.

## 4. Dynamics

The reason for the specific tuning of the scaling parameter $\beta$ in order to achieve non-triviality may be understood in terms of the dynamics of the theory. From (10), (11) and (12) we can calculate the form of the propagators for the $\varphi_i$, $\chi$ and $\rho$ fields at $\varphi_i=0$, $\chi=\chi_0(0)=m^2$, $\rho=\rho_0(0)$. If we write the propagators symbolically as in figure 3(*a*), then, to leading order in the $1/N$ expansion, we may form a composite scalar field $\chi'$ whose interaction with the $\varphi$-fields is identical to that of the $\chi$- field, by summing all the diagrams of the form in figure 3(*b*). We find in the large-$N$ limit

$$D_{\chi'}(k^2)=\frac{-(2/N)[(\beta/\hbar)\delta(0)/G^2(m^2)-\tfrac{1}{2}\lambda_0]}{\hbar B(k^2,m^2)[(\beta/\hbar)\delta(0)/G^2(m^2)-\tfrac{1}{2}\lambda_0]-1} \tag{34}$$

where

$$B(k^2,m^2)=\int đp[(k-p)^2+m^2]^{-1}[p^2+m^2]^{-1} \tag{35}$$

and it can be shown that for general values of $\beta$ the denominator diverges. In this case, the $\chi'$-field does not propagate and since this field mediates all interactions the theory is trivial.

† This form generalises that used by Rivers and shows that the canonical mass-gap equation should also result from the pseudo-free theory in four dimensions.

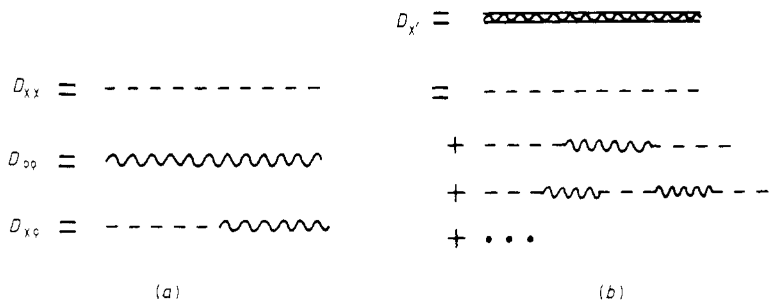


**Figure 3.** Representing the $\chi$–$\chi$, $\rho$–$\rho$ and $\chi$–$\rho$ propagators as in ($a$), we can form a composite field $\chi'$ (whose interaction with the $\varphi$-fields is identical to that of the $\chi$-field) as in ($b$), each term of which is of the same order in $1/N$.

However, for the specific 'tuned' value of $\beta$, the divergences in the denominator cancel and the $\chi'$-fields propagate, leading to a non-trivial interaction between the $\varphi$-fields.

Since the mass-gap equations are identical to their canonical counterparts when $\beta$ is tuned, and since the effective potential is uniquely determined by the mass-gap equation, the $\chi'$-field can be understood in terms of bound-states and resonances of the $\varphi$-fields as in Rembiesa (1978) and Abbott *et al* (1976).

## 5. Conclusion

We have shown that the work of Rivers on the scale-covariant pseudo-free theory is directly generalisable to a theory with a $\lambda_0\varphi^4$-type interaction, and that the theory may be non-trivial in $d=4$ and $d=5$ dimensions for a specific choice of the scaling parameter $\beta$. Indeed, for this specific 'tuned' value of $\beta$, the scale-covariant $\lambda_0(\varphi^2)^2$ theory appears identical to the pseudo-free theory which in turn appears identical to the canonical $\lambda_0(\varphi^2)^2$ theory at the leading order. It thus seems that the 'hard-core' effects of scale-covariant quantisation act as a $\lambda\varphi^4$-type interaction.

Our results still suffer from the problems of the $1/N$ expansion, and no conclusive answer as to the non-triviality of the scale-covariant theory can be made. However, it appears that if a non-trivial theory does exist, it will require a very specific tuning of the scaling parameter. This tuned value of $\beta_\Lambda$ is dependent on $\lambda_0$ and in $d=4$ dimensions it is given by

$$\beta_\Lambda=\lambda_0\frac{\hbar S_4}{2}-\frac{2}{1/g+\ln(M^2/\Lambda^2)} \qquad (36)$$

to leading order in $1/N$.

In a recent paper (Ogielski 1983), a Monte Carlo study of the four-dimensional scale-covariant $\lambda_0\varphi^4$ theory was carried out. The results suggest that for the values of $\beta$ and $\lambda_0$ in the study, the theory is trivial. This, however, is not surprising as no attempt has been made to tune $\beta_\Lambda$. Our results imply that for a general value of $\beta_\Lambda$, the theory is indeed trivial. The possibility of a 'tuned' Monte Carlo study is now being considered and results will be given elsewhere.

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